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# Domain walls on the brane

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## Abstract

We show that all branes admit worldvolume domain wall solutions. We find one class of solutions for which the tension of the brane changes discontinuously along the domain wall. These solutions are not supersymmetric. We argue that there is another class of domain wall solutions which is supersymmetric. A particular case concerns supersymmetric domain wall solutions on IIB D-5- and NS-5-branes. © 1998 Published by Elsevier Science B.V. All rights reserved.

## 1. Introduction

The investigation of the worldvolume solitons of various branes have given new insights in the understanding of the properties of M-theory and string theory. From the bulk perspective, the worldvolume solitons on a brane have the interpretation as the intersection region of two branes or the boundary of a brane ending on another [1–3]. As such the worldvolume solitons themselves have a brane interpretation. Many such worldvolume solitons have already been found [4–7] and perhaps the most notable one is the 0-brane soliton on a D-brane which can be thought as the boundary of a fundamental string ending on it [7].

A special class of solitons are domain walls. One example of a domain wall is the D-8-brane of massive IIA supergravity which separates spacetime in two disconnected regions [8,9]. The D-8-brane is associated with a ten-form field strength whose dual is the spacetime cosmological constant. The value of the ten-form at spatial infinity is the charge of the

domain wall at the corresponding asymptotic region. The main property of the D-8-brane solution is that the cosmological constant changes discontinuously at the position of the brane. We expect that domain walls on the worldvolume of branes will have similar properties. In particular, (i) there should be a field that changes discontinuously at the position of the wall and (ii) the domain wall will separate the worldvolume of the brane into two regions. In this work we shall investigate charged domain walls which are associated with a  $(p+1)$ -form field strength on the worldvolume of the  $p$ -brane. There are two kinds of worldvolume  $(p+1)$ -form field strengths: (i) one class of  $(p+1)$ -form field strengths is associated to the tension of branes in the scale invariant formulation [10,11] of the worldvolume brane actions<sup>1</sup>; (ii) another class of such worldvolume forms is due to intersections of a  $p$ -brane with a

<sup>1</sup> The substitution of a  $(p+1)$ -form field strength for the tension of a brane is similar to the substitution of a ten-form field strength for the ten-dimensional cosmological constant.

$q$ -brane on a  $(p-1)$ -brane. As it has been argued in [3] for such intersections to occur the worldvolume action of the  $p$ -brane should have a  $(p+1)$ -form field strength in its spectrum. If the intersecting brane configuration preserves a proportion of the supersymmetry of the bulk, the associated worldvolume domain wall will preserve the same proportion of supersymmetry.

In this paper, we first present domain wall solutions for all  $p$ -branes which are associated with the first class of  $(p+1)$ -form field strengths mentioned above, i.e. the ones that correspond to their tension in the scale invariant formulation of the worldvolume actions. We shall show that these domain wall solutions break all the supersymmetry of the bulk. We shall verify this both by a direct computation and by an argument based on intersecting branes. Next, we shall examine domain wall solutions on  $p$ -branes associated with the second class of  $(p+1)$ -form field strengths. We shall be mainly concerned with the intersection of a IIB D-5-brane and a IIB NS-5-brane on a 4-brane. From either the perspective of the D-5-brane or the perspective of the NS-5-brane the associated worldvolume soliton is a domain wall. We shall argue that the worldvolume actions of the D-5-brane and the NS-5-brane should contain a 6-form field strength due to the intersection which belongs to the second class and is different from the one that describes their tensions. Using an analogy with the standard BI field we shall give the kappa-symmetry transformations of the actions involving this new 6-form field strength up to terms linear in the fields. We shall present a solution for this domain wall and we shall show that it is supersymmetric.

This letter is organized as follows: In section two, we give the domain wall solutions associated with the brane tensions and show that they are not supersymmetric. In section three, we give two arguments for the existence of supersymmetric domain walls; one based on a T-duality chain and the other based on the six-dimensional (1,1) worldvolume supersymmetry algebra. In section four, we discuss such supersymmetric domain wall solutions for the IIB NS-5-brane and IIB D-5-brane and in section five we present our conclusions. In a separate appendix, following [12], we list all the central charges of the supersymmetry algebra corresponding to the IIB NS-

5-brane and relate every soliton on the worldvolume of the IIB NS-5-brane to a possible intersection involving a IIB NS-5-brane in the bulk.

## 2. Domain walls and scale invariant actions

The domain walls that we shall investigate do not involve the standard Born-Infeld fields. This allows us to treat all cases in a uniform way. Let  $X$  be the embedding map of a  $p$ -brane into ten-dimensional Minkowski spacetime. We introduce a Lagrange multiplier  $V$  and write the worldvolume action of the  $p$ -brane as [10,11]

$$I = \int d^{p+1}u \left( \frac{1}{2V} \det(g_{\mu\nu}) - T^2 V \right), \quad (1)$$

where  $T$  is a parameter which is identified with the tension of the  $p$ -brane <sup>2</sup>,  $\{u^\mu; \mu = 0, \dots, p\}$  are the worldvolume coordinates and

$$g_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N \eta_{MN} \quad (2)$$

is the induced metric on the  $p$ -brane ( $\eta_{MN}$  is the ten-dimensional Minkowski spacetime metric). Using the field equation of the Lagrange multiplier, it is straightforward to show that this action is equivalent to the usual Nambu-Goto action. Next, we shall follow [11] and postulate the action

$$I = \int d^{p+1}u \left( \frac{1}{2V} \left[ \det(g_{\mu\nu}) + 4(\tilde{F})^2 \right] \right), \quad (3)$$

where

$$F_{\mu_1, \mu_2, \dots, \mu_{p+1}} = (p+1) \partial_{[\mu_1} A_{\mu_2, \dots, \mu_{p+1}]} \quad (4)$$

is a  $(p+1)$ -form field strength and

$$\tilde{F} = \frac{1}{(p+1)!} \epsilon^{\mu_1, \dots, \mu_{p+1}} F_{\mu_1, \dots, \mu_{p+1}} \quad (5)$$

is the Poincaré dual of  $F$ .

The field equations of the above action are

$$\begin{aligned} \det(g_{\mu\nu}) + 4(\tilde{F})^2 &= 0, \quad \partial_\mu (V^{-1} \tilde{F}) = 0, \\ \partial_\mu (V^{-1} \det(g_{\lambda\rho}) g^{\mu\nu} \partial_\nu X^M) &= 0. \end{aligned} \quad (6)$$

It has been shown in [11] that the field equations of

<sup>2</sup> Strictly speaking,  $T = 1/\sqrt{2}$  times the physical tension of the brane.

the action (1) are the same as those of the action (3). To see this, we solve the second equation in (6) by setting

$$V^{-1}\tilde{F} = \frac{T}{\sqrt{2}}, \quad (7)$$

where  $T$  is the tension of the  $p$ -brane as in (1).

To find the domain wall worldvolume soliton on a  $p$ -brane associated with  $F$ , we shall adopt a similar point of view to that adopted for finding the D-8-brane solution of the massive IIA supergravity theory [9] and allow the tension  $T$  to be *piece-wise constant*. Next, we choose the static gauge,  $\{X^M\} = \{(u^\mu, Y^i); \mu = 0, \dots, p, i = p+1, \dots, 9\}$ . We take  $z = u^p$  to be the coordinate transverse to the domain wall and write the ansatz

$$Y^i = Y^i(z), \quad V = V(z), \quad \tilde{F} = (-1)^p \partial_z W(z). \quad (8)$$

Using this ansatz, the last two field equations of (6) can be rewritten as

$$\partial_z(V^{-1}\partial_z W) = 0, \quad \partial_z(V^{-1}\partial_z Y^i) = 0. \quad (9)$$

One solution of these equations is

$$V^{-1}\partial_z W = \begin{cases} w_1 & \text{for } -\infty < z < 0 \\ w_2 & \text{for } 0 < z < +\infty \end{cases}, \quad (10)$$

and

$$V^{-1}\partial_z Y^i = \begin{cases} y_1^i & \text{for } -\infty < z < 0 \\ y_2^i & \text{for } 0 < z < +\infty \end{cases}, \quad (11)$$

where  $w_1, w_2, y_1^i, y_2^i$  are real constants. Substituting (10) and (11) into the first field equation of (6) we find

$$V(z) = \begin{cases} \pm \frac{1}{\sqrt{4w_1^2 - |y_1|^2}} & \text{for } -\infty < z < 0 \\ \pm \frac{1}{\sqrt{4w_2^2 - |y_2|^2}} & \text{for } 0 < z < +\infty \end{cases}, \quad (12)$$

where  $|y_1|^2 = \delta_{ij}y_1^i y_1^j$  and  $|y_2|^2 = \delta_{ij}y_2^i y_2^j$ . It remains to solve for  $W$  and  $Y$ . If we allow  $V$  to be piece-wise constant, then the Eqs. (9) will become

differential equations with distributional coefficients. We wish to avoid this, so we shall take  $V$  to be constant everywhere which leads to the condition

$$4w_2^2 - |y_2|^2 = 4w_1^2 - |y_1|^2 \quad (13)$$

for the constants of the solution. The Eqs. (9) then imply that  $W, Y^i$  are harmonic functions which we shall take to be continuous but not differential at  $z = 0$ , i.e.

$$W(z) = \pm \frac{1}{\sqrt{4w_1^2 - |y_1|^2}} \begin{cases} w_1 z & \text{for } -\infty < z < 0 \\ w_2 z & \text{for } 0 < z < +\infty \end{cases}, \quad (14)$$

and

$$Y^i = \pm \frac{1}{\sqrt{4w_1^2 - |y_1|^2}} \begin{cases} y_1^i z & \text{for } -\infty < z < 0 \\ y_2^i z & \text{for } 0 < z < +\infty \end{cases}. \quad (15)$$

A more general solution can be found by setting

$$Y^i = H_2^i, \quad \tilde{F} = (-1)^p \partial_z H_1, \quad (16)$$

where  $H_1, H_2^i$  are harmonic functions of the real line. For  $V$  to be a constant, as required,  $H_1, H_2^i$  should have the same centres and their first derivatives should satisfy (13) at every centre. The domain walls are located at the centres of the harmonic functions. Using the relation between the tension and  $\tilde{F}$ , see (7), we find that

$$T = (-1)^p \sqrt{2} V^{-1} \partial_z H_1. \quad (17)$$

It is clear from this that the tension of the  $p$ -brane is different at the two sides of a domain wall.

These domain walls are not supersymmetric. This can be verified by a direct computation using the kappa-symmetry transformations of the scale invariant brane actions as given in [11], i.e.

$$\delta\theta = V^{-1} (2\tilde{F} + \Gamma_{(0)})\kappa, \quad (18)$$

where  $\kappa$  is the transformation parameter and  $\Gamma_{(0)}$  is the product structure associated with the  $p$ -brane (see e.g. [13] where it has been denoted with  $\Gamma'_{(0)}$ ). The above kappa-symmetry transformation will suffice for our purposes since the standard BI field

vanishes for our configurations. Using a modification of the argument in [13], we find that the supersymmetry condition is

$$(2\tilde{F} - \Gamma_{(0)})\epsilon = 0, \quad (19)$$

where  $\epsilon$  is the supersymmetry parameter. We shall not give the details of this computation for all  $p$ -branes but we shall consider the case of the D-2-brane. Let us suppose that the D-2-brane lies in the direction 0,1,2 and that there is one non-vanishing transverse scalar along the direction 3. Substituting the domain wall ansatz in (19), we find that

$$(2\partial_z W - \Gamma_0 \Gamma_1 (\Gamma_2 + \partial_z Y \Gamma_3))\epsilon = 0. \quad (20)$$

For the domain wall to be supersymmetric, the solutions of above equation should be *constant* spinors; the constancy condition is required from the bulk (supergravity) killing spinor equations for Minkowski background<sup>3</sup> It is clear then that  $\epsilon$  should satisfy the conditions

$$\Gamma_0 \Gamma_1 \Gamma_2 \epsilon = \epsilon, \quad \Gamma_0 \Gamma_1 \Gamma_3 \epsilon = \epsilon. \quad (21)$$

However there are no non-trivial solutions to the above equations since the two product structures anticommute and therefore do not have common solutions.

An alternative way to show that such a domain wall soliton is not supersymmetric is to use the bulk picture of such solitons as arising from the intersection of two branes. We shall present our argument for the case that only one of the transverse scalars  $Y$  is non-zero. We remark that for these solutions  $\tilde{F}$  does not vanish. Let us suppose that there is a supersymmetric intersecting brane configuration associated with the above domain wall solitons. Since the soliton changes the tension of the  $p$ -brane discontinuously, it implies that the intersection will involve two  $p$ -branes of the same kind intersecting on a  $(p-1)$ -brane. However no such supersymmetric bulk intersecting brane configurations exist which preserve 1/4 or less of the supersymmetry of the bulk. This argument is consistent with the example that we have presented above. For this, observe that the product structures of (21) are those of two planar

D-2-branes lying in the directions 0,1,2 and 0,1,3, respectively, and therefore intersecting at a string. There is no such supersymmetric configuration.

### 3. Domain walls and supersymmetry

So far we have investigated non-supersymmetric domain walls. Here we shall present two arguments indicating that there exist supersymmetric domain walls as well. One of the arguments is based on T-duality and the other is based on the properties of the worldvolume supersymmetry algebra.

To give some examples of supersymmetric domain walls, we first extend the T-duality chain of [14] as follows:

$$(0|1_D, 5_S)_B \xleftrightarrow{T} (1|2_D, 5_S)_A \xleftrightarrow{T} (2|3_D, 5_S)_B \xleftrightarrow{T} (3|4_D, 5_S)_A \xleftrightarrow{T} (4|5_D, 5_S)_B \xleftrightarrow{T} (5|6_D, 5_S)_A. \quad (22)$$

The notation that we are using,  $(k|p, q)$ , denotes a  $p$ -brane and a  $q$ -brane intersecting on a  $k$ -brane and the various subscripts are self-explanatory. As suggested in [14], from the perspective of NS-5-branes the various brane solitons lying on the intersections of two branes or as boundaries of branes ending on branes transform like D-branes under IIA and IIB T-duality. For the first four intersections of the T-duality chain a corresponding supersymmetric worldvolume soliton has been constructed (for the IIB 0-brane soliton, see [7]; for the IIA string and three-brane solitons, see [5,6]). The next intersection of the T-duality chain naturally suggests the existence of a 4-brane, i.e. domain wall solution, on the NS-5-brane. Since T-duality preserves supersymmetry, we expect the domain wall solution to be supersymmetric. This domain wall solution will be analyzed in the next section.

Let us consider the case  $(4|5_D, 5_S)_B$  in more detail. From either the perspective of the IIB D-5-brane or the perspective of the IIB NS-5-brane the 4-brane soliton is a worldvolume domain wall. From this D-brane perspective, the above duality chain suggests that there should be a *supersymmetric* domain wall solution on every D-brane.

An alternative way to argue for the existence of supersymmetric domain walls on the IIB NS-5-brane

<sup>3</sup> We remark though that (20) has always solutions if  $\epsilon$  is allowed to be *locally constant*.

and IIB D-5-brane is to use their worldvolume supersymmetry algebra as in [15,12]. The worldvolume theory of both IIB NS-5- and D-5-branes is described by the reduction to six dimensions of the  $N = 1$  ten-dimensional super-Maxwell multiplet. The associated six-dimensional (1,1) supersymmetry algebra including central charges is given by

$$\begin{aligned}\{Q_{\alpha i}, Q_{\beta j}\} &= \epsilon_{ij}(\gamma^\mu)_{\alpha\beta}(P + Z)_\mu + (X^+)_{(\alpha\beta)(ij)}, \\ \{Q_{\alpha i}, \tilde{Q}_{\beta j}\} &= (\gamma^{\mu\nu})_\alpha^\beta Y_{\mu\nu, ij} + W_{ij}\delta_\alpha^\beta, \\ \{\tilde{Q}_{\alpha i}, \tilde{Q}_{\beta j}\} &= \epsilon_{ij}(\gamma^\mu)^{\alpha\beta}(P - Z)_\mu + (X^-)_{(\alpha\beta)(ij)},\end{aligned}\quad (23)$$

where  $\{Q_{\alpha i}, \tilde{Q}_{\beta j}; \alpha = 1, \dots, 4; i, j = 1, \dots, 2\}$  are the sixteen supersymmetry charges,  $P$  is the energy momentum,  $Y_{ij}$  are two-form charges,  $Z$  is a one-form charge,  $W_{ij}$  are scalar charges and  $X^+, X^-$  are the self-dual and the anti-self-dual parts of a three-form charge<sup>4</sup>. Following [12], we identify the central charges of the supersymmetry algebra with the various brane solitons on the worldvolume of the associated branes (for a full list and the relation to all bulk intersections involving the IIB NS-5-brane, see the appendix). In particular, from the perspective of the NS-5-brane the charge that corresponds to the domain wall soliton is the ‘electric’ component of the two-form central charge  $Y_{ij}$  which is a vector of  $SO(4)$ . It is straightforward to show that this soliton is supersymmetric using the supersymmetry algebra.

#### 4. Supersymmetric domain walls on IIB 5-branes

In this section we shall investigate the supersymmetric domain wall solutions on the IIB NS-5-brane. The related problem of domain wall solutions on the IIB D-5-brane is S-dual to this. In order to have domain wall solutions on the IIB NS-5-brane, the worldvolume action of the theory should include a 6-form field strength  $f$ <sup>5</sup>. However such a worldvol-

ume action for the NS-5-brane is not known<sup>6</sup>. The presence of the 6-form field strength  $f$  on the worldvolume of the NS-5-brane is due to the intersection of the IIB NS-5-brane with the IIB D-5-brane. This is reminiscent to the presence of the standard BI field in the worldvolume action of D-branes because fundamental strings end on D-branes<sup>7</sup>. Using this analogy, we write an action for the IIB NS-5-brane up to quadratic terms for the fields which includes  $f$  as

$$I = \frac{T}{2} \int d^6 u \left( \eta^{\mu\nu} \partial_\mu Y^\dagger \partial_\nu Y^j + |f|^2 \right) \quad (24)$$

in the static gauge. Following the analogy, the kappa-symmetry transformations of this new action should be similar to those of D-branes including the contribution from the (standard) BI field. So we write the kappa-symmetry transformation of the spacetime fermions  $\theta$  as<sup>8</sup>

$$\delta\theta = \left[ 1 + (1 + i\tilde{f}\sigma_2)\Gamma_{(0)} \right] \kappa, \quad (25)$$

where  $\kappa$  is the parameter,  $\Gamma_{(0)}$  is the supersymmetry projector associated with the IIB NS-5-brane and  $\tilde{f}$  is the dual of  $f$ . In the above expression, we keep only terms linear in the fields. The presence of  $\sigma_2$  in the kappa-symmetry transformations interchanges the two chiral ten-dimensional fermions  $\kappa$ . To explain this, we use the fact that  $f$  is associated with the intersection of the D-5-brane with the NS-5-brane and that the supersymmetry projector of the D-5-brane interchanges the two chiral ten-dimensional fermions  $\kappa$  which serve as the parameters in the kappa-symmetry transformations of the D-5-brane [17–19]. The associated supersymmetry condition is

$$(1 + i\tilde{f}\sigma_2)\Gamma_{(0)}\epsilon = \epsilon \quad (26)$$

where  $\epsilon$  is the supersymmetry parameter.

Next let us suppose that the IIB NS-5-brane is located in the directions 0,1,2,3,4,5 and the direction

<sup>4</sup> The greek indices are spinor indices while the Roman ones transform under the automorphism group  $SU(2) \times SU(2)$  of the algebra which is the spin group of  $SO(4)$ .

<sup>5</sup> Since we are interested in supersymmetric domain walls we assume that this is a different 6-form field strength from that of the scale invariant action considered in section two.

<sup>6</sup> This is related to the problem of how to construct  $SL(2, \mathbb{R})$  covariant actions for IIB (p,q) 5-branes.

<sup>7</sup> For the case of IIB (p,q) 1-branes a  $SL(2, \mathbb{R})$  covariant action, containing *two* worldvolume vector fields, has been constructed [16].

<sup>8</sup> Note that this kappa-symmetry rule differs from the one used in Section 2.

transverse to the domain wall is  $z = u^5$ . To find a supersymmetric domain wall solution, we use the ansatz

$$\tilde{f} = -\partial_z W, \quad Y = Y^6(z). \quad (27)$$

We remark that from the bulk perspective the D-5-brane lies in the directions 0,1,2,3,4,6. Substituting this ansatz in the supersymmetric condition above and keeping only terms linear in the fields, we find that supersymmetry is preserved if

$$\partial_z W = \partial_z Y \quad (28)$$

provided that

$$\begin{aligned} \Gamma_0 \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5 \epsilon^1 &= \epsilon^1, \\ \Gamma_0 \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5 \epsilon^2 &= -\epsilon^2, \\ \Gamma_0 \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_6 \epsilon^1 &= \epsilon^2. \end{aligned} \quad (29)$$

These are the bulk supersymmetry projectors of the IIB NS-5-brane and the IIB D-5-brane. In particular, the first two projectors are associated with the NS-5-brane and the latter one is associated with the D-5-brane. The supersymmetry preserved is 1/4 of the bulk and so 1/2 of the worldvolume. The field equations imply that  $W$  is a harmonic function  $H$  of  $z$  and therefore the solution can be written as

$$\tilde{f} = -\partial_z H, \quad Y = H(z). \quad (30)$$

It is straightforward now to find the supersymmetric domain wall on the IIB D-5-brane due to the intersection with the NS-5-brane.

It remains to comment about the possibility of domain walls on the other D-branes involved in the T-duality chain (22). We remark that in some cases, like the one involving the D-string and IIB NS-5-brane, the D-brane ends on the NS-5-brane. In such a case, it is not clear to us that there should be a domain wall from the perspective of the D-brane<sup>9</sup>.

## 5. Conclusions

The worldvolume actions of  $p$ -branes admit domain wall solutions which are associated with vari-

ous worldvolume  $(p+1)$ -form field strengths. First, we have investigated a class of non-supersymmetric domain walls where the worldvolume  $(p+1)$ -form field strength is dual to the tension of the  $p$ -brane. Next, we have given one example of supersymmetric domain walls on the IIB NS-5- and D-5-branes. The T-duality argument presented in section three suggests the existence of many more supersymmetric domain wall solutions which we plan to investigate elsewhere.

One application of the existence of domain walls on the worldvolume of branes is to indicate that there are processes where the brane is separated into two pieces along the wall. Such processes will involve the tension of the brane and therefore the non-supersymmetric domain walls.

Another implication of our work is that the worldvolume  $(p+1)$ -form field strength, needed in the construction of the domain wall solution, suggests that the maximal supersymmetric Maxwell multiplet in D-dimensions admits an extension by a D-form field strength. Such a field does not introduce additional propagating degrees of freedom and similar extensions have been considered before for other multiplets [20]. This follows from the fact that all D-branes have a kappa-symmetric scale invariant action<sup>10</sup>. We shall demonstrate this for the ten-dimensional Maxwell theory.

The other cases follow by dimensional reduction. Let  $A$  be the one-form Maxwell gauge potential,  $F = dA$ ,  $\chi$  be the spacetime Majorana-Weyl fermion partner of  $A$  and  $B$  be a 9-form gauge potential,  $G = dB$ . An action for this multiplet is

$$S = \int d^{10}x |F|^2 + \bar{\chi} \not{D} \chi + |\tilde{G}|^2, \quad (31)$$

where  $\tilde{G}$  is the dual of  $G$ . The supersymmetry transformations, leaving this action invariant, are given by

$$\begin{aligned} \delta A_\mu &= \bar{\epsilon} \Gamma_\mu \chi, \quad \delta \chi = \Gamma^{\mu\nu} F_{\mu\nu} \epsilon + \tilde{G} \eta, \\ \delta B_{\mu_1, \dots, \mu_9} &= \bar{\eta} \Gamma_{\mu_1, \dots, \mu_9} \chi. \end{aligned} \quad (32)$$

where  $\epsilon$  is the standard supersymmetry parameter and  $\eta$  is a new parameter that has the same chirality

<sup>9</sup> A three-brane domain wall on the D-4-brane worldvolume has been discussed in [6].

<sup>10</sup> We thank Paul Townsend for a discussion on this point.

as  $\epsilon$ . Note that the supersymmetry algebra closes up to a central charge transformation. It would be interesting to investigate the properties of this modified Maxwell multiplet further.

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## Appendix A

In this appendix, following [12], we list all the central charges of the supersymmetry algebra corresponding to the IIB NS-5-brane and relate every soliton on the worldvolume of the IIB NS-5-brane to a possible intersection involving a IIB NS-5-brane in the bulk.

As we have already mentioned in section three, the supersymmetry algebra (23) has two three-form charges  $X^+, X^-$ , a two-form charge  $Y^{ij}$ , two one-form charges  $P, Z$  and a scalar charge  $W_{ij}$ . These charges transform under  $SO(4)$  as follows:  $X^+, X^-$  transform as self-dual, anti-self-dual two forms, re-

Table 2

Brane intersections. This table relates every  $p$ -form ( $p'$ -form) worldvolume soliton of the previous table to two different brane intersections. The two cases correspond to the cases  $t$  and  $t'$  in Table 1.

Soliton	$t$	$t'$
0	$(0 5_S, 1_D)$	$(0 5_S, 3_D)$
1	$(1 5_S, 1_F)$	$(1 5_S, 5_S)$
2	$(2 5_S, 3_D)$	$(2 5_S, 5_D)$
3	$(3 5_S, 5_S)$	$(3 5_S, KK)$
4	$(4 5_S, 5_D)$	$(4 5_S, 7_D)$
5	$(5 5_S, KK)$	$(5 5_S, 9_S)$

spectively,  $Y^{ij}, W_{ij}$  transform as a vector and  $Z$  transforms as a scalar. We summarize in the Table 1 the worldvolume solitons associated with these central charges as well as the number of non-zero transverse scalars associated with them. We do this for the NS-5-brane. Table 2, for the D-5-brane, is similar.

The brane intersections associated with these worldvolume solitons are summarized in Table 2.

The intersections involving the D-5-brane can be summarized in a similar way.

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Table 1

Worldvolume solitons. This table relates every central charge of the supersymmetry algebra (23) to a  $p$ -form ( $p'$ -form) worldvolume soliton. The numbers  $t$  and  $t'$  indicate the number of transverse directions from the bulk perspective.

Charge	$p$	$p'$	$t$	$t'$
$X^+$	3	-	2	-
$X^-$	-	3	-	2
$Y$	2	4	1	3
$Z$	1	5	0	4
$W$	0	-	1	3



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